Decoherence of coupled electron spins via nuclear spin dynamics in quantum dots

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In double quantum dots, the exchange interaction between two electron spins renormalizes the excitation energy of pair flips in the nuclear spin bath, which in turn modifies the non-Markovian bath dynamics. As the energy renormalization varies with the static Overhauser field mismatch between the quantum dots, the electron singlet-triplet decoherence resulting from the bath dynamics depends on sampling of nuclear spin states from an ensemble, leading to the transition from superexponential decoherence in single-sample dynamics to power-law decay under ensemble averaging. In contrast, the decoherence of a single electron spin in one dot is essentially the same for different choices of the nuclear spin configuration.

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I. INTRODUCTION

Decoherence draws a boundary between the microscopic quantum world and the macroscopic classical world. It is also a main obstacle in quantum technologies such as quantum computation. Thus, both for understanding crossover from the quantum to the classical world[1–3] and for exploiting quantum coherence of large systems,[4] it is desirable to comprehend how decoherence develops with scaling up the size of a system. The very initial step toward such a purpose is to examine the difference between a two-level system (the simplest quantum object, called a qubit in quantum computation) and two coupled ones. For a system in a Markovian bath (which has broad-band fluctuation), the decoherence is described by the Lindblad formalism. For a system in a non-Markovian bath, there are indications of nontrivial scaling behavior of decoherence,[5–8] such as the nonadditive decoherence in multiple baths.[5] In this paper, we study the decoherence of a composite quantum object in a non-Markovian mesoscopic bath based on a paradigmatic system in mesoscopic physics and quantum information science,[9–11] namely, two electron spins in double quantum dots (QDs).

In III-V semiconductor QDs, where a high degree of spin control has been achieved,[10–13] the dominant decoherence channel at low temperature is the hyperfine interaction with the lattice nuclear spins, which serves as an ideal realization of the general spin bath model.[14] The dynamics of the mesoscopic nuclear spin bath in a QD is conditioned on the state of the electron spin in contact with the bath.[15–17] The conditional evolution of the nuclear spins establishes the electron-nuclear entanglement that causes the electron spin decoherence. When the electron spin is disturbed, the nuclear bath dynamics is altered. For example, the nuclear spin evolution can be shepherded by a structured sequence of electron spin flips so that the electron is disentangled from the bath and, as a result, the lost coherence is recovered.[16] Besides external control, the disturbance may also be due to interaction with another quantum object in proximity, such as the exchange interaction $J_{\mathrm{ex}}\hat{\mathbf{S}}_1\cdot\hat{\mathbf{S}}_2$ between two electron spins in coupled QDs. Intuitively speaking, the disturbance due to the exchange interaction may be viewed as precessing of the electron spins about each other. For a more rigorous treatment, we should first diagonalize the electron spin Hamiltonian including the exchange interaction and then study the bath dynamics and the decoherence in the electron eigenstate basis.

To demonstrate the most essential physics, we consider only the decoherence between the singlet state $|S\rangle$ and the unpolarized triplet state $|T_0\rangle$ (by assuming that the polarized triplet states $|T_\pm\rangle$ are well split off by a large external magnetic field). For free-induction decay (FID) in ensemble experiments, the singlet-triplet (S-T) coherence time is limited to several nanoseconds by the inhomogeneous broadening of the static nuclear Overhauser field mismatch.[18,19] The inhomogeneous dephasing can be eliminated through spin-echo techniques; thus, the most relevant decoherence mechanism is the dynamical electron-nuclear entanglement, established by the bath evolution conditioned on the electron spin state. The interaction inside the bath is essential to the decoherence since, for noninteracting nuclear spin baths, the pure dephasing caused by the hyperfine interaction would be totally eliminated from spin-echo signals.[15–17,20]

In this paper, we study the entanglement-induced decoherence between $|S\rangle$ and $|T_0\rangle$ using a realistic interacting nuclear spin bath model. The theoretical method is presented in Sec. II. The results for the S-T decoherence in FID and under concatenated spin-echo control are given in Sec. III. The conclusion is given in Sec. IV.

II. MODEL AND THEORY

We consider a gate-defined symmetric double-dot structure similar to those used in Refs. 10, 11, 18, and 19 under a perpendicular magnetic field. Assuming a large on-site Coulomb energy and gate voltages supporting one electron in each dot, the low-lying two electron states consist of the singlet state $|S\rangle=(1/\sqrt{2})(d_{1\uparrow}^{\dagger}d_{2\downarrow}^{\dagger}-d_{1\downarrow}^{\dagger}d_{2\uparrow}^{\dagger})|0\rangle$ and the triplet states $|T_+\rangle=d_{1\uparrow}^{\dagger}d_{2\uparrow}^{\dagger}|0\rangle$, $|T_-\rangle=d_{1\downarrow}^{\dagger}d_{2\downarrow}^{\dagger}|0\rangle$, and $|T_0\rangle=(1/\sqrt{2})(d_{1\uparrow}^{\dagger}d_{2\downarrow}^{\dagger}+d_{1\downarrow}^{\dagger}d_{2\uparrow}^{\dagger})|0\rangle$, where $d_{j\uparrow}^{\dagger}$ ($d_{j\downarrow}^{\dagger}$) ($j=1,2$) creates one spin-up (spin-down) electron in the dot-localized state $\varphi_j(\mathbf{r})$ in the jth QD and $|0\rangle$ is the state with no electrons in the double QDs. The Hamiltonian for the electron-nuclear spin system is

$$\hat{H}=\Omega_e\sum_{j=1,2}\hat{S}_j^z+J_{\mathrm{ex}}\hat{\mathbf{S}}_1\cdot\hat{\mathbf{S}}_2+\sum_{j=1,2}\hat{\mathbf{S}}_j\cdot\hat{\mathbf{h}}_j+\hat{H}_N, \tag{1}$$

where $\hat{\mathbf{S}}_1$ and $\hat{\mathbf{S}}_2$ are spin operators of the two electrons, $\Omega_e=g^*\mu_B B$ is the Zeeman splitting, J_{ex} is the exchange energy (due to virtual interdot tunneling and direct Coulomb exchange), $\hat{\mathbf{h}}_j=\Sigma_n a_{j,n}\hat{\mathbf{I}}_{j,n}$ $(j=1,2)$ is the nuclear Overhauser field (due to contact hyperfine interaction) with $a_{j,n}$ the hyperfine coefficient for the nth nuclear spin $\hat{\mathbf{I}}_{j,n}$ at the jth dot, and $\hat{H}_N$ is the Hamiltonian of the interacting nuclear spin bath, including the nuclear Zeeman energy and the intrinsic nuclear spin interaction.[15,17] Under a large magnetic field $(\Omega_e \gg J_{\rm ex})$, the unpolarized states $|S\rangle$ and $|T_0\rangle$ are well separated in energy from the polarized triplet states $|T_\pm\rangle$. The off-diagonal hyperfine interaction (terms with $\hat{S}_j^x$, $\hat{S}_j^y$) couples $|S\rangle$ and $|T_0\rangle$ to $|T_\pm\rangle$, which, however, has negligible effect under a strong external magnetic field $(\Omega_e \ggg a_{j,n})$.[21] The Overhauser field mismatch between the two dots, $\hat{\Delta}\equiv\hat{h}_1^z-\hat{h}_2^z$, causes the flip-flop between the singlet state $|S\rangle$ and the triplet state $|T_0\rangle$. The effective S-T Hamiltonian is

$$\hat{H}_{\rm eff}=J_{\rm ex}\frac{|T_0\rangle\langle T_0|-|S\rangle\langle S|}{2}+\hat{\Delta}\frac{|T_0\rangle\langle S|+|S\rangle\langle T_0|}{2}. \qquad (2)$$

The Overhauser field mismatch $\hat{\Delta}$ causes longitudinal T_1 relaxation. For a relatively large exchange splitting (e.g., $J_{\rm ex}\gtrsim 10\hat{\Delta}$), the T_1 process is suppressed,[18,19] but virtual S-T flips induce a self-energy correction. To incorporate this effect, we introduce dressed singlet and triplet;[22] then, the Hamiltonian in Eq. (2) is formally diagonalized as $\sqrt{J_{\rm ex}^2+\hat{\Delta}^2}(|T_0\rangle\langle T_0|-|S\rangle\langle S|)/2$, which serves as a self-energy operator in the dressed basis. For a given initial nuclear spin configuration $|\mathcal{I}\rangle$, we divide $\hat{\Delta}$ into the static part $\Delta_{\mathcal{I}}\equiv\langle\mathcal{I}|\hat{\Delta}|\mathcal{I}\rangle$ and the small quantum fluctuation $\hat{\delta}_{\mathcal{I}}\equiv\hat{\Delta}-\Delta_{\mathcal{I}}$. By expanding $\sqrt{J_{\rm ex}^2+\hat{\Delta}^2}$ around its static value $E_{\rm S\text{-}T}\equiv\sqrt{J_{\rm ex}^2+\Delta_{\mathcal{I}}^2}$ to second order of $\hat{\delta}_{\mathcal{I}}$, $\hat{H}_{\rm eff}$ is separated into the mean-field part,

$$\hat{H}_{\rm MF}=E_{\rm S\text{-}T}\frac{|T_0\rangle\langle T_0|-|S\rangle\langle S|}{2}, \qquad (3)$$

and the part containing the small quantum fluctuation of the Overhauser field mismatch,

$$\hat{H}_{\rm QF}\approx\hat{H}_Z(|T_0\rangle\langle T_0|-|S\rangle\langle S|), \qquad (4)$$

where

$$\hat{H}_Z=\frac{\Delta_{\mathcal{I}}}{2E_{\rm S\text{-}T}}\hat{\delta}_{\mathcal{I}}+\frac{\hat{\delta}_{\mathcal{I}}^2}{4E_{\rm S\text{-}T}}. \qquad (5)$$

Here, $E_{\rm S\text{-}T}$ (the renormalized S-T splitting) and $\hat{H}_Z$ characterize the static and dynamic self-energy corrections, respectively. The second term in $\hat{H}_Z$ is typically much smaller than the first one since $\langle\hat{\delta}_{\mathcal{I}}\rangle_{\rm rms}\ll\langle\Delta_{\mathcal{I}}\rangle_{\rm rms}$, but it would be the leading term for nuclear spin configurations with vanishing $\Delta_{\mathcal{I}}$. Now, the Hamiltonian of the singlet-triplet system and the nuclear spin bath is reduced to

$$\hat{H}_{\rm S\text{-}T}=\hat{H}_+|T_0\rangle\langle T_0|+\hat{H}_-|S\rangle\langle S|, \qquad (6)$$

with

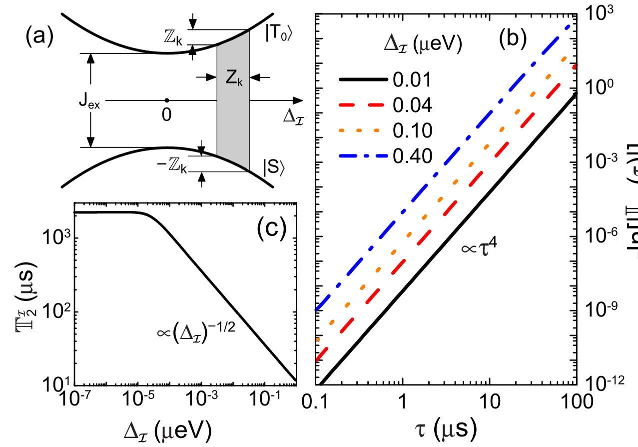

FIG. 1. (Color online) (a) The singlet and triplet energies as functions of the Overhauser field mismatch $\Delta_{\mathcal{I}}$ and the (renormalized) excitation energy of a nuclear spin pair flip Z_k $(\mathbb{Z}_k)$. (b) FID of the S-T coherence for various initial nuclear spin states, indicated by $\Delta_{\mathcal{I}}$. (c) FID decoherence time $\mathbb{T}_2^{\mathcal{I}}$ as a function of $\Delta_{\mathcal{I}}$.

$$\hat{H}_\pm=\hat{H}_N\pm E_{\rm S\text{-}T}/2\pm\hat{H}_Z. \qquad (7)$$

Such a block-diagonal Hamiltonian induces no T_1 relaxation but only pure S-T dephasing. The static self-energy correction $\pm E_{\rm S\text{-}T}/2$ causes inhomogeneous dephasing in ensemble FID measurements. The dynamic self-energy correction $\pm\hat{H}_Z$ renormalizes the nuclear spin excitation spectrum and leads to entanglement-induced dephasing.

The role of the exchange interaction in the decoherence may be disclosed in a semiclassical spectral diffusion picture.[23] Let us denote the local Overhauser fields for the nuclear spin configuration $|\mathcal{I}\rangle$ by the electron Zeeman energies $\Omega_1^{\mathcal{I}}$ and $\Omega_2^{\mathcal{I}}$ in dots 1 and 2, respectively. The pairwise nuclear spin flip-flops cause the fluctuation of the Overhauser fields, and therefore a random phase of the electron spins. For a single electron spin in one QD, the Zeeman energy change due to the kth pair flip is $2Z_k$. With exchange interaction, the S-T splitting $E_{\rm S\text{-}T}$ varies with the Overhauser field mismatch $\Delta_{\mathcal{I}}\equiv\Omega_1^{\mathcal{I}}-\Omega_2^{\mathcal{I}}$ [see Fig. 1(a)], so the S-T splitting change due to the kth pair flip is

$$2\mathbb{Z}_k\approx(2Z_k)\frac{\partial E_{\rm S\text{-}T}}{\partial\Delta_{\mathcal{I}}}\approx 2Z_k\frac{\Delta_{\mathcal{I}}}{E_{\rm S\text{-}T}}. \qquad (8)$$

Thus, the exchange interaction modifies the energy fluctuation (or spectral diffusion) responsible for the electron spin decoherence.

In a full quantum mechanical picture,[15–17,24,25] the S-T decoherence is caused by the electron-nuclear entanglement, established during the evolution of the nuclear spin state predicated on the electron states. Suppose the electrons are initially in a superposition state $\alpha|S\rangle+\beta|T_0\rangle$ and the nuclear spin state $|\mathcal{I}\rangle$ is one randomly chosen from the thermal ensemble $\hat{\rho}_N=e^{-\beta\hat{H}_N}=\Sigma_{\mathcal{I}}P_{\mathcal{I}}|\mathcal{I}\rangle\langle\mathcal{I}|$. For temperature much greater than intrinsic nuclear spin interaction energy $(\sim 10^{-9}$ K), the nuclear spin density matrix $\hat{\rho}_N$ has no off-diagonal coherence and we can choose $|\mathcal{I}\rangle$ as an eigenstate of the noninteracting nuclear spin bath. Starting from the initial state $(\alpha|S\rangle+\beta|T_0\rangle)\otimes|\mathcal{I}\rangle$, the conditional nuclear spin evolution $|\mathcal{I}\rangle\rightarrow|\mathcal{I}^{\pm}(\tau)\rangle\equiv e^{-i\hat{H}_{\pm}\tau}|\mathcal{I}\rangle$ establishes an entangled state $\alpha|S\rangle\otimes|\mathcal{I}^{-}(\tau)\rangle+\beta|T_0\rangle\otimes|\mathcal{I}^{+}(\tau)\rangle$. The S-T coherence for a single sample of nuclear spin configuration $|\mathcal{I}\rangle$ is $\mathbb{L}_{\mathcal{I}}(\tau)=\langle\mathcal{I}^{-}(\tau)|\mathcal{I}^{+}(\tau)\rangle=e^{-iE_{\text{S-T}}\tau}|\langle\mathcal{I}^{-}(\tau)|\mathcal{I}^{+}(\tau)\rangle|$. In ensemble dynamics, the signal is to be averaged by $\mathbb{L}(\tau)=\Sigma_{\mathcal{I}}P_{\mathcal{I}}\mathbb{L}_{\mathcal{I}}(\tau)$. The static fluctuation $\Delta_{\mathcal{I}}$ results in an inhomogeneous dephasing,[18]

$$\sum_{\mathcal{I}}P_{\mathcal{I}}e^{-iE_{\text{S-T}}\tau}\sim\frac{1}{\sqrt{1+i(\tau/\tau_0)}}\tag{9}$$

(for a Gaussian distribution of $\Delta_{\mathcal{I}}$ with standard variance Γ), with a nanosecond decoherence time $\tau_0\approx J_{\text{ex}}/\Gamma^2$,[18,19] much faster than the entanglement-induced decoherence $\mathbb{L}_{\mathcal{I}}(\tau)$ arising from the quantum fluctuation $\hat{\delta}_{\mathcal{I}}$ in single-sample dynamics, which will be later shown to have the microsecond time scale.

To calculate the nuclear spin evolution $|\mathcal{I}^{\pm}(\tau)\rangle$, we employ the pair-correlation approximation in which all possible pair flips from the initial configuration $|\mathcal{I}\rangle$ are taken as independent of each other. The pair-correlation approximation is justified for a mesoscopic nuclear spin bath with a sufficiently random configuration where, within the decoherence time scale, the pair flips that occurred are much fewer than the pairs available to be flipped, and therefore have little probability to be in neighborhood of each other or to be correlated.[15–17,24–27] A pair flip is characterized by a transition strength due to the off-diagonal nuclear spin interaction $B_k=\langle\mathcal{I}|\hat{H}_N|\mathcal{I},k\rangle$, an energy cost due to the diagonal nuclear spin interaction $D_k=\langle\mathcal{I},k|\hat{H}_N|\mathcal{I},k\rangle-\langle\mathcal{I}|\hat{H}_N|\mathcal{I}\rangle$, and a hyperfine-energy cost $\pm\mathbb{Z}_k=\pm(\langle\mathcal{I},k|\hat{H}_Z|\mathcal{I},k\rangle-\langle\mathcal{I}|\hat{H}_Z|\mathcal{I}\rangle)$ for the triplet and the singlet state, respectively, where $|\mathcal{I},k\rangle$ denotes the nuclear spin state after the kth pair flip. An independent pair flip is mapped to be a spin-1/2 pseudospin $\hat{\mathbf{s}}_k$ precessing about a pseudofield $\boldsymbol{\chi}_k^{\pm}\equiv(2B_k,0,D_k\pm\mathbb{Z}_k)$, initially pointing to the down direction.[15–17] The nuclear Hamiltonian in the pseudospin representation is $\hat{H}_{\pm}\approx\pm E_{\text{S-T}}/2+\Sigma_k\boldsymbol{\chi}_k^{\pm}\cdot\hat{\mathbf{s}}_k$.

In the quantum picture, the exchange interaction not only renormalizes the static self-energy correction $\pm E_{\text{S-T}}/2$, and hence influences the inhomogeneous dephasing, but also modifies the hyperfine-energy cost of a nuclear pair flip from $\pm Z_k$ (for spin-up and spin-down states of uncorrelated electrons, respectively) to $\pm\mathbb{Z}_k\approx\pm(\Delta_{\mathcal{I}}Z_k/E_{\text{S-T}}+Z_k^2/E_{\text{S-T}})$ (for the triplet and singlet states, respectively). The bath dynamics itself is altered when noninteracting quantum objects in the bath are replaced by interacting ones. In particular, through the dependence of the bath fluctuation on the static Overhauser field mismatch $\Delta_{\mathcal{I}}$, the dynamics of nuclear spins in one dot is affected by the state of the nuclear spins in the other dot. Therefore, the resultant S-T decoherence time varies with sampling of the nuclear spin configuration $|\mathcal{I}\rangle$ from the ensemble $\hat{\rho}_N$. This, as will be shown later, leads to a transition from superexponential decoherence to power-law decay upon ensemble averaging. On the contrary, the decoherence of a single electron spin in a QD is essentially independent of the static Overhauser field.[15,17]

In numerical evaluation, we take a symmetric GaAs double-dot structure with height of 6 nm, Fock-Darwin radius of 70 nm for a parabolic confinement potential, and center-to-center separation of 137 nm under a perpendicular magnetic field $B=1$ T at temperature $T=1$ K. The zinc-blende lattice structure and realistic nuclear spin interactions are included.[17] To model the isotope disorder of ^{69}Ga and ^{71}Ga, we randomly place ^{69}Ga and ^{71}Ga on cation sites according to their respective natural abundance. We have verified that different random generators give essentially the same results. For a specific ^{69}Ga and ^{71}Ga distribution, the variance of the Overhauser field mismatch $\Gamma\equiv\sqrt{\langle\hat{\Delta}^2\rangle-\langle\hat{\Delta}\rangle^2}$ due to thermal fluctuation is given by

$$\Gamma=\sqrt{\sum_{j,n}a_{j,n}^2[\langle(\hat{I}_{j,n}^z)^2\rangle-\langle\hat{I}_{j,n}^z\rangle^2]},\tag{10}$$

where $\langle\hat{O}\rangle\equiv\text{Tr}(\hat{\rho}_N\hat{O})=\Sigma_{\mathcal{I}}P_{\mathcal{I}}\langle\mathcal{I}|\hat{O}|\mathcal{I}\rangle$ denote the thermal average of the expectation value of an operator $\hat{O}$. Direct calculation yields $\Gamma\approx0.12$ μeV, corresponding to an inhomogeneous dephasing time $T_2^*=\sqrt{2}/\Gamma\approx8$ ns. The exchange energy $J_{\text{ex}}\approx-1$ μeV is determined with the Hund-Mulliken method,[28] consistent with experimental values.[10]

III. RESULTS AND DISCUSSIONS

A. Free-induction decay

Within the pair-correlation approximation, the single-sample S-T coherence in FID,

$$\mathbb{L}_{\mathcal{I}}(\tau)=e^{-iE_{\text{S-T}}\tau}\prod_k|\langle\psi_k^-(\tau)|\psi_k^+(\tau)\rangle|,\tag{11}$$

is given by the overlap of pseudospin wave functions $|\psi_k^{\pm}(\tau)\rangle=e^{-i\boldsymbol{\chi}_k^{\pm}\cdot\hat{\mathbf{s}}_k\tau}|\downarrow_k\rangle$. The short-time behavior $\mathbb{L}_{\mathcal{I}}(\tau)\approx\exp[-iE_{\text{S-T}}\tau-\Sigma_k\delta_k^2(\tau)/2]$ (for $\mathbb{Z}_k\tau\ll1$) is described by the distance $\delta_k(\tau)\equiv|\mathbf{S}_k^+(\tau)-\mathbf{S}_k^-(\tau)|=\sqrt{1-|\langle\psi_k^-(\tau)|\psi_k^+(\tau)\rangle|^2}$ between pseudospin expectation values $\mathbf{S}_k^{\pm}(\tau)=\langle\psi_k^{\pm}(\tau)|\hat{\mathbf{s}}_k|\psi_k^{\pm}(\tau)\rangle$, initially both pointing in the down direction $\mathbf{S}_k^{\pm}(0)=-\mathbf{e}_z/2$ but precessing about different pseudofields $\boldsymbol{\chi}_k^{\pm}$. Since the pseudofield difference $\boldsymbol{\chi}_k^+-\boldsymbol{\chi}_k^-=2\mathbb{Z}_k\mathbf{e}_z$ characterizing the relative motion between $\mathbf{S}_k^{\pm}(\tau)$ is collinear with the initial value $\mathbf{S}_k^{\pm}(0)$, the distance $\delta_k(\tau)$ initially increases with a linearly increasing speed $d\delta_k(\tau)/d\tau\propto\tau$, leading to τ^2 increase of $\delta_k(\tau)$ and, consequently, a quartic exponential decay $\mathbb{L}_{\mathcal{I}}(\tau)\approx\exp(-iE_{\text{S-T}}\tau-\Sigma_kB_k^2\mathbb{Z}_k^2\tau^4/2)\equiv e^{-iE_{\text{S-T}}\tau-(\tau/\mathbb{T}_2^{\mathcal{I}})^4}$ (for $\mathbb{Z}_k\tau\ll1$). The decoherence time $\mathbb{T}_2^{\mathcal{I}}\propto(\mathbb{Z}_k)^{-1/2}\propto(\Delta_{\mathcal{I}}/E_{\text{S-T}})^{-1/2}$, varying with sampling of the nuclear spin configuration from the ensemble $\hat{\rho}_N$. For comparison, the decoherence of a single electron spin in a QD,[29] except for a trivial global phase factor related to the inhomogeneous dephasing, is essentially independent of choice of the initial state,[15–17] since the excitation energy of a nuclear pair flip there, $Z_k=\langle\mathcal{I},k|\hat{h}_j^z/2|\mathcal{I},k\rangle-\langle\mathcal{I}|\hat{h}_j^z/2|\mathcal{I}\rangle$, is independent of $|\mathcal{I}\rangle$. Also, the

S-T decoherence time is longer than the single spin decoherence time by a factor of $\sqrt{E_{\text{S-T}}/\Delta_{\mathcal{I}}}$ because of the reduction of the excitation energy, and therefore the flip rates of the pair flips.

Figure 1(b) shows the FID decoherence for several nuclear spin initial states $|\mathcal{I}\rangle$ randomly chosen from the thermal ensemble. The suppression of the decoherence with decreasing the Overhauser field mismatch $(\Delta_{\mathcal{I}})$ is evident. The dependence of the decoherence time on the static Overhauser field mismatch $\mathbb{T}_2^{\mathcal{I}} \propto (\Delta_{\mathcal{I}}/E_{\text{S-T}})^{-1/2}$ is verified in Fig. 1(c). Note that for vanishing mismatch $\Delta_{\mathcal{I}} \to 0$, the energy cost $\mathbb{Z}_k$ vanishes in the leading order of Z_k and the second-order correction $\mathbb{Z}_k \approx Z_k^2/J_{\text{ex}}$ [see Eq. (5)], resulting in a saturation of the decoherence time at a large value. In ensemble FID, the static fluctuation $\Delta_{\mathcal{I}}$ induced inhomogeneous dephasing dominates over the quantum fluctuation $\hat{\delta}_{\mathcal{I}}$ induced single-sample decoherence, so, below, we study the ensemble-averaged coherence in spin-echo configurations where the inhomogeneous dephasing is eliminated.

B. Single-pulse Hahn echo

The static random phase between the singlet and triplet states can be eliminated by spin echo. Note that in the present case, the S-T flip $|S\rangle \leftrightarrow |T_0\rangle$ instead of the single spin flip $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$ should be applied. In experiments, the S-T flip may be realized by an impulsive change of the Overhauser field mismatch via, e.g., applying a pulse of local magnetic field to one of the double dots,[11] which causes a π-phase shift of one of the two electron spins.

In the single-pulse Hahn echo (τ-π-τ echo), the single-sample S-T coherence at the echo time 2τ is

$$\mathbb{L}_{\mathcal{I}}^{(1)}(2\tau) = \prod_k |\langle \downarrow_k|(\hat{U}_k^{(1),-})^\dagger \hat{U}_k^{(1),+}|\downarrow_k\rangle|, \tag{12}$$

where $\hat{U}_k^{(1),\pm} \equiv e^{-i\chi_k^{\mp}\cdot\hat{\mathbf{s}}_k\tau} e^{-i\chi_k^{\pm}\cdot\hat{\mathbf{s}}_k\tau}$. The short-time behavior (for $\tau \ll \mathbb{Z}_k^{-1}$) is $\mathbb{L}_{\mathcal{I}}^{(1)}(2\tau) \approx \exp(-2\Sigma_k B_k^2 \mathbb{Z}_k^2 \tau^4) \equiv e^{-(2\tau/\mathbb{T}_H^{\mathcal{I}})^4}$ with the single-sample decoherence time $\mathbb{T}_H^{\mathcal{I}} = (\Sigma_k B_k^2 \mathbb{Z}_k^2/8)^{-1/4} = \sqrt{2}\mathbb{T}_2^{\mathcal{I}}$. The ensemble dynamics is studied by averaging over a large number of samples from a Gaussian distribution of the static Overhauser field mismatch. With the approximation $E_{\text{S-T}} \approx J_{\text{ex}}$, the ensemble-averaged result is analytically obtained for $\tau \ll \mathbb{Z}_k^{-1}$,

$$\mathbb{L}^{(1)}(2\tau) \approx [1 + (2\tau/\mathbb{T}_H)^4]^{-1/2}, \tag{13}$$

with a power-law decay profile, where the ensemble decoherence time $\mathbb{T}_H = \mathbb{T}_H^{\mathcal{I}}|_{\Delta_{\mathcal{I}}=\sqrt{2}\Gamma}$, i.e., the decoherence time for a nuclear spin configuration with the static Overhauser field mismatch equal to $\sqrt{2}$ times the standard variance. Such a transition from a superexponential decay in single-sample dynamics to a power-law decay in ensemble dynamics is shown in Fig. 2. In contrast, the echo signal of a single electron spin in a QD is unchanged by ensemble averaging, i.e., $L^{(1)}(2\tau) = L_{\mathcal{I}}^{(1)}(2\tau)$. As emphasized in the Introduction, for theories with noninteracting nuclear spin bath, either of the single-sample coherence $\mathbb{L}_{\mathcal{I}}^{(1)}(2\tau)$ and $L_{\mathcal{I}}^{(1)}(2\tau)$ or the ensemble-averaged ones $\mathbb{L}^{(1)}(2\tau)$ and $L^{(1)}(2\tau)$ would not show any decay at all.

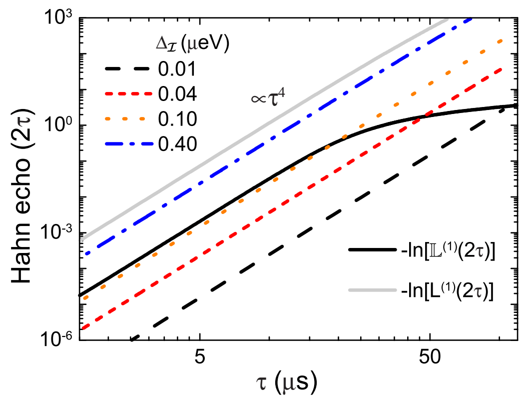

FIG. 2. (Color online) Hahn echo signal $(-\ln[\mathbb{L}_{\mathcal{I}}^{(1)}(2\tau)])$ for various nuclear spin configurations $|\mathcal{I}\rangle$ as indicated by $\Delta_{\mathcal{I}}$. The solid black line shows the ensemble-averaged coherence $(-\ln[\mathbb{L}^{(1)}(2\tau)])$, compared to the echo signal $(-\ln[L^{(1)}(2\tau)])$ of a single spin in a QD of the same size (solid gray line).

C. Concatenated pulse control

We now study the S-T decoherence under concatenated control which is designed to preserve the coherence.[16,17,30] The coherence preserved after the mth order concatenated pulse sequence $\mathbb{L}_{\mathcal{I}}^{(m)}(\tau_m)$ is obtained by substituting $\hat{U}_k^{(m),\pm}$ for $\hat{U}_k^{(1),\pm}$ in Eq. (12), where $\tau_m \equiv 2^m\tau$, and $\hat{U}_k^{(m),\pm}$ is recursively defined as $\hat{U}_k^{(m-1),\mp}\hat{U}_k^{(m-1),\pm}$ for $m>1$. The short-time profile (for $\tau \ll \mathbb{Z}_k^{-1}$) is $\mathbb{L}_{\mathcal{I}}^{(m)}(\tau_m) \approx e^{-(\tau_m/\mathbb{T}_{2,m}^{\mathcal{I}})^{2m+2}}$, with the decoherence time $\mathbb{T}_{2,m}^{\mathcal{I}} \sim 2^{m(m+3)/2(m+1)} [Z_k B_k^m (\Delta_{\mathcal{I}}/E_{\text{S-T}})]^{-1/(m+1)}$.[17] The τ^{2m+2} exponential profile is a general feature of spin decoherence under the mth order concatenation, independent of the specific bath Hamiltonian (see Ref. 17 for details). Again, the ensemble average leads to a power-law decay

$$\mathbb{L}^{(m)}(\tau_m) = [1 + (\tau_m/\mathbb{T}_{2,m})^{2m+2}]^{-1/2}, \tag{14}$$

with $\mathbb{T}_{2,m} = \mathbb{T}_{2,m}^{\mathcal{I}}|_{\Delta_{\mathcal{I}}=\sqrt{2}\Gamma}$. In contrast, for a single electron spin, the ensemble averaging has negligible effect, i.e., $L^{(m)}(\tau_m) = L_{\mathcal{I}}^{(m)}(\tau_m) \approx e^{-(\tau_m/T_{2,m})^{2m+2}}$, where the decoherence time $T_{2,m}$ is shorter than the S-T decoherence time $\mathbb{T}_{2,m}$ by a factor $\sim (J_{\text{ex}}/\Gamma)^{1/(m+1)}$. Figure 3 compares the S-T decoherence to the single spin decoherence, showing the suppression of the decoherence and the crossover to a power-law decay due to the coupling between the electron spins.

D. Precession-driven decoherence profile transition

The single-sample S-T coherence $\mathbb{L}_{\mathcal{I}}^{(m)}(\tau_m)$ in spin-echo configurations has a negative exponential factor $\propto \mathbb{Z}_k^2 \tau^{2m+2} \sim (\Delta_{\mathcal{I}}^2/J_{\text{ex}}^2)\tau^{2m+2}$. In comparison, the single-sample S-T coherence $\mathbb{L}_{\mathcal{I}}(\tau)$ for FID has an imaginary exponential factor $-iE_{\text{S-T}}\tau \sim -i\Delta_{\mathcal{I}}^2\tau/(2J_{\text{ex}})$. Such similar dependence on the static Overhauser field mismatch leads to the interesting result that after ensemble averaging over the Gaussian distribution of $\Delta_{\mathcal{I}}$, the entanglement-induced S-T decoherence and the inhomogeneous S-T dephasing both have a power-law profile [cf. Eqs. (14) and (9)]. Namely, in different experimental configurations (spin-echo versus FID), the quantum

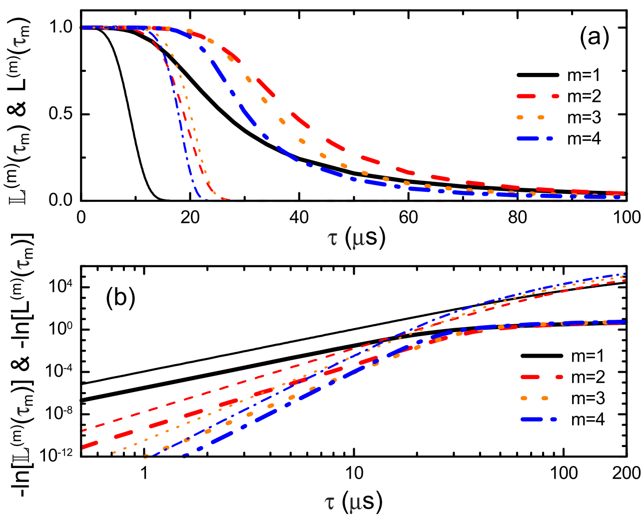

FIG. 3. (Color online) (a) Ensemble-averaged coherence under concatenated control for the S-T decoherence in two coupled dots [$\mathbb{L}^{(m)}(\tau_m)$, as thick lines] and for the single spin decoherence in one dot [$L^{(m)}(\tau_m)$, as thin lines], where m indicates the concatenation level. (b) Logarithmic plot of (a).

fluctuation $\hat{\delta}_{\mathcal{I}}$ and the static fluctuation $\Delta_{\mathcal{I}}$ of the nuclear Overhauser field mismatch produce similar decoherence profiles, although the $\pi/4$ phase shift (for $\tau \gg \tau_0$) in inhomogeneous dephasing[18] is absent for the entanglement-induced decoherence. We emphasize, however, that the nature of the decoherence, the underlying mechanisms, and the relevant time scale are qualitatively different.

The renormalized S-T decoherence may be intuitively understood in terms of the precession of the electron spins driven by the exchange interaction between them. The precession, which is rapid as compared to the hyperfine flip-flop, eliminates the first-order ($\propto \Delta_{\mathcal{I}}$) electron-nuclear hyperfine interaction. Consequently, the remaining second-order interaction ($\propto \Delta_{\mathcal{I}}^2/J_{\text{ex}}$) leads to enhanced coherence time and the ensemble average leads to a power-law decay. This precession-driven decoherence suppression and decay profile transition are an extension of the motional narrowing picture, previously discovered in the context of nuclear magnetic resonance spectroscopy and D'yakonov-Perel' spin relaxation.[31,32] Very recently, this phenomenon has been demonstrated experimentally[33] for the inhomogeneous dephasing in single spin Rabi rotation, where the spin precession is driven by an external Rabi field.

IV. CONCLUSION

In conclusion, the exchange interaction between two electron spins in double QDs modifies the nuclear spin bath dynamics through renormalizing the pair-flip excitation energy. As the renormalized excitation energy varies with the static Overhauser field mismatch between the two dots, the nuclear spin dynamics in one dot becomes dependent on the nuclear spin state in the other dot, regardless of the nonexistence of interdot nuclear spin interaction in the considered situation. Consequently, the S-T decoherence due to the electron-nuclear entanglement depends on the choice of the nuclear spin configuration from the ensemble, leading to a power-law decay of ensemble-averaged coherence, in contrast with the superexponential decoherence of a single electron spin which is insensitive to sampling of the nuclear spin ensemble. The dependence of the S-T decoherence on the static Overhauser field mismatch may be observed by tuning the mismatch with an inhomogeneous external field. The exchange interaction also enhances the S-T decoherence time by suppressing the fluctuation in the nuclear spin bath.

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[22] From now on, singlet or $|S\rangle$ (triplet or $|T_0\rangle$) denotes dressed singlet (dressed triplet). This leads to small visibility loss in the ensemble coherence (Ref. 15) since the Overhauser field mismatch $\hat{\Delta}$ is much smaller than the exchange energy $J_{\rm ex}$.

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